Ecology of Cognitive Radio Ad Hoc Networks

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***Abstract*—The success of cognitive radio networks lies on whether secondary users (SUs) follow the sharing rule that do not disturb the existing primary users (PUs). However, SUs may not behave in the desirable way as long as their own transmissions are successful. In this case, PUs may suffer heavy interference and system is deteriorated. Via evolutionary game modeling, this letter studies the dynamics of cognitive radio ad hoc networks against misbehaved SUs due to sensing errors or selfish nature. Our study suggests that cognitive radio network is fragile to the misbehaved SUs if incentives for the successful secondary transmissions are high. Consequently, all other SUs evolve to dishonest behaviors leading to network collapse.**

***Index Terms*—cognitive radio, ecology, evolutionary game, misbehavior, selfish behavior.**

I. INTRODUCTION

RECENTLY, cognitive radio (CR) received lots of attentions due to its potential to improve spectrum efficiency. Unlicensed secondary users (SUs) sense surrounding environment and adapt their operations around those of the licensed primary users (PUs) to opportunistically exploit available resources while limiting their interference with PUs. In *underlay* sharing paradigm [1], the interference imposed by secondary transmission coexisting with primary transmission shall be below a predefined constraint. In *interweave* paradigm, a zero-interference rationale is adopted, that is, SU can not interfere with the PU at all. This implies that SUs only use the spectrum which is not temporarily used by PUs and are obligated to evacuate the spectrum upon sensing primary transmission.

Obviously, spectrum sensing is an essential operation of CRs to ensure the success of spectrum sharing. However, the sensing result may be corrupted and unreliable due to crucial issues (such as hidden terminal problems, fast fading, shadowing, and noise power uncertainty) and the *miss detection* may incur harmful interference to PUs. Even if the sensing results are correct, selfish SUs aiming to maximize their own utilities may be always transmitting despite sharing rules [2]. As shown in Fig. 1, those SUs causing unacceptable interference to PU due to inadvertent malfunction, miss detection, or selfish behavior are considered as *misbehaved SUs* in this letter.

In cognitive radio ad hoc networks (abbreviated as CRNs) without centralized control, misbehaviors are difficult to be caught, punished and deterred. Without the threats, selfish SUs will no longer behave as "hit and run radios" [3] but unscrupulously make their own transmissions despite mutual interference, which deteriorates the system performance dramatically. In this case, whether the sharing rules can be enforced is of critical importance. Substantial works have been done to model the competition among the (mis)behaved SUs by using game theory [2], [4], [5]. However, little research effort was made to analyze to what extent these misbehaviors can impact the survivability of other behaved SUs.

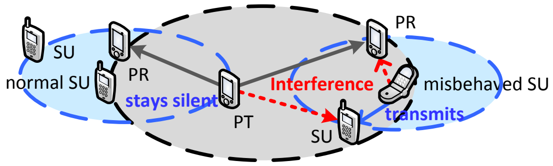

Fig. 1. Normal and misbehaved SUs in most worrisome case for PU: when the primary transmitter (PT) can cause little interference to the SU, but the SU can cause major interference to the primary receiver (PR).

Inspired by ecological biology, SUs with different behaviors could be considered as species with different access strategies. The original problem can be interpreted as how those misbehaved species evolve and whether the behaved species are extinct when misbehavior is inevitable. By modeling misbehaved SUs as mutations, evolutionary game [6]–[8] is utilized to analyze the dynamics of access strategy via natural selection, that is, the resilience of CRN against misbehaved SUs is investigated.

The incentive of successful transmission plays a central role in our analysis. We adopt stochastic geometry to model the interference among PUs and SUs, which affects the probability of successful transmission. When there is no incentive for SU to deviate from the sharing rule, *self-enforcement* [4] is achieved and misbehaved species extinct. If incentive for successful secondary transmission is high, behaved SUs will no long play honestly. Consequently, all users suffer heavy interference and the network collapses. This study provides a criterion on designing incentives for successful SU transmission, and operator could adopt the results to avoid vital outbreak and enhance the resilience of CRN.

II. SYSTEM MODEL

We consider the scenario that a CRN coexists with PUs by utilizing the same spectrum. Considering the cumulative effects of interference, outage occurs if the received SINR is below the receiver sensitivity due to too many simultaneous transmissions. Based on our previous efforts [9], we leverage stochastic geometry to characterize the behavioral features of retransmission and medium access in CRN. The spatial distributions of primary transmitters (PTs) and SUs are assumed to follow homogeneous Poisson point processes (PPPs)

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with densities λ_{PT} and λ_{SU}, respectively. Each PT has transmission power P_{PT} and a dedicated primary receiver (PR) located at a fixed distance r_{PT} with an arbitrary direction. The spatial distribution of PRs also forms a PPP with the same density λ_{PT} correlated with that of PTs. Due to the stationary characteristics, the interference measured by a typical PR could represent the interference seen by other PRs. To prevent from incurring harmful interference to sensed PRs and SUs, in each time slot each SU adopts slotted ALOHA protocol to independently access the spectrum with probability p, where p is the parameter of i.i.d. Bernoulli random variables, $B_i(p)$.

Let $\Phi_{PT} = \{X_i\}$ ($\Phi_{SU} = \{Y_i\}$) denote the locations of the PTs (SUs). For a typical PR, the received SINR is

$$SINR_{PR} = \frac{\mathcal{G}_{PT} P_{PT} r_{PT}^{-\alpha}}{N + I_{SU} + I_{PT}}, \tag{1}$$

where $\mathcal{G}_{PT}$ is the channel gain of the desired link which is exponentially distributed with unit mean (i.e, slow flat Rayleigh fading channel), α is the path loss exponent, N is the noise power level, $\Phi_{SU}(\widetilde{p}) = \{Y_i : B_i(\widetilde{p}) = 1\}$ is the set of active SUs, $I_{SU} = \sum_{Y_i \in \Phi_{SU}(p)} \mathcal{G}_{Y_i} P_{SU} \|Y_i\|^{-\alpha}$ is the interference from SUs to the typical PR, $I_{PT} = \sum_{X_i \in \Phi_{PT}} \mathcal{G}_{X_i} P_{PT} \|X_i\|^{-\alpha}$ is the interference from other PTs to the typical PR, $\|\cdot\|$ is the distance to the typical PR located at the origin, P_{SU} is the transmission power of SU and $\mathcal{G}_{X_i}$ ($\mathcal{G}_{Y_i}$) is the channel gain which is exponentially distributed with unit mean. The received SINR of a typical SU is

$$SINR_{SU} = \frac{\mathcal{G}_{SU} P_{SU} r_{SU}^{-\alpha}}{N + I_{SU} + I_{PT}}, \tag{2}$$

where $\mathcal{G}_{SU}$ denotes the channel gain of the desired link which is exponentially distributed with unit mean, and r_{SU} is the transmission distance.

With the cognitive feature, SU adjusts its access probability to ensure the sufficient operation of primary transmissions, i.e., the outage probability of PR satisfies $\mathbb{P}(SINR_{PR} \geq \eta_{PR}) \geq 1 - \epsilon_{PR}$, where η_{PR} is the PR sensitivity and ϵ_{PR} is the maximum outage probability. In the interweave paradigm, only intra-system interference is considered (i.e., $I_{SU} = 0$ in (1) and $I_{PT} = 0$ in (2)). Regarding underlay paradigm, the maximum permissible density of active SUs with the outage constrains of PUs is $\widetilde{\lambda}_{SU} = \frac{-\ln(1-\epsilon_{PR}) - (\eta_{PR}/P_{PT} r_{PT}^{-\alpha})N}{r_{PT}^{2} \eta_{PR}^{\omega} K_\alpha} P_{SU}^{-\omega}$, $K_\alpha = \frac{2\pi^2}{\alpha \sin(2\pi/\alpha)}$, and $\omega = \frac{2}{\alpha}$ [9]. The spectrum utilization can be enhanced without causing unsustainable interference to PUs if every SU adopts the active probability $\widetilde{p} = \frac{\widetilde{\lambda}_{SU}}{\lambda_{SU}}$.

III. EVOLUTIONARY ACCESS GAME

Upon each transmission, every SU accesses the spectrum with an adjusted probability $p \in [0, 1]$ according to its previous spectrum payoff. Denote the normalized population density of SUs with respective to a given p as $\theta(p)$ and population p.d.f. of SUs as θ. The behaved SUs adopt initial access probability $\widetilde{p}$ while misbehaved SUs transmit for sure (i.e., $p = 1$). For the typical SU, the transmission is successful if the received SINR is no less than the receiver sensitivity, and the access utility function with access probability p_j is defined as

$$U_j(\theta) = \begin{cases} \delta, \text{ if } SINR_{SU} \geq \eta_{SU}, \\ \nu, \text{ if } SINR_{SU} < \eta_{SU}, \end{cases}$$

where ν is the cost of unsuccessful transmission, and δ is the incentive of successful transmission.

Assuming the cost of staying silent is zero and following the derivation in [9], the average payoff per transmission with access probability p_j when the access population is θ becomes

$$\begin{aligned}
\bar{U}_j(p_j|\theta) &= p_j \left\{\delta \mathbb{P}(SINR_{SU} \geq \eta_{SU}) - \nu \mathbb{P}(SINR_{SU} < \eta_{SU})\right\} \\
&= p_j \left\{(\delta + \nu)\mathbb{P}(SINR_{SU} \geq \eta_{SU}) - \nu\right\} \\
&= p_j \left\{(\delta + \nu)\mathbb{P}\left[\mathcal{G}_{SU} \geq \frac{\eta_{SU}(N + I_{PT} + I_{SU})}{P_{SU} r_{SU}^{-\alpha}}\right] - \nu\right\} \\
&= p_j \left\{(\delta + \nu)\exp\left(-\frac{\eta_{SU} N}{P_{SU} r_{SU}^{-\alpha}}\right) \mathbb{E}\left[\exp\left(-\frac{\eta_{SU} I_{PT}}{P_{SU} r_{SU}^{-\alpha}}\right)\right]\right. \\
&\quad \left. \cdot\ \mathbb{E}\left[\exp\left(-\frac{\eta_{SU} I_{SU}}{P_{SU} r_{SU}^{-\alpha}}\right)\right] - \nu\right\} \\
&= p_j \left\{(\delta + \nu)\exp\left(-\frac{\eta_{SU} N}{P_{SU} r_{SU}^{-\alpha}}\right) \exp\left\{-\left[\lambda_{PT}\left(\frac{P_{PT}}{P_{SU}}\right)^{\omega}\right.\right.\right. \\
&\quad \left.\left.\left. +\ \lambda_{SU} \int_0^1 p\theta(p)dp\right] r_{SU}^2 \eta_{SU}^{\omega} K_\alpha\right\} - \nu\right\} \\
&\triangleq p_j \left\{(\delta + \nu) A \exp(-B f(\theta)) - \nu\right\},
\end{aligned} \tag{3}$$

where $\lambda_{SU} \int_0^1 p\theta(p)dp$ represents the density of active SUs. The increase of active SUs may deteriorate the receiver sensitivity and has a negative impact on the access payoff. Please note that in the interweave paradigm, the utility function is obtained by setting $I_{PT} = 0$ ($\lambda_{PT} = 0$) in (3). Let $\bar{U}(x, y) = \int_0^1 x \bar{U}_j(p|y)dp$ denote the payoff with access population x and experienced population y. Initially (without mutants), the average payoff is

$$\bar{U}(\theta, \theta) = \int_0^1 \theta(p) \bar{U}_j(p|\theta)dp. \tag{4}$$

Suppose τ ($\tau \in (0, 1)$) portion of SUs become mutants and access the spectrum with a different population probability distribution $\hat{\theta}$, the experienced access population is $\tau\hat{\theta} + (1 - \tau)\theta$ and the average payoff of non-mutants becomes

$$\begin{aligned}
\bar{U}_n &= \bar{U}(\theta, \tau\hat{\theta} + (1 - \tau)\theta) \\
&= \int_0^1 \theta(p)p[(\delta + \nu)A \exp(-Bf(\tau\hat{\theta} + (1 - \tau)\theta)) - \nu]dp.
\end{aligned} \tag{5}$$

Similarly, the average payoff of mutants becomes

$$\begin{aligned}
\bar{U}_m &= \bar{U}(\hat{\theta}, \tau\hat{\theta} + (1 - \tau)\theta) \\
&= \int_0^1 \hat{\theta}(p)p[(\delta + \nu)A \exp(-Bf(\tau\hat{\theta} + (1 - \tau)\theta)) - \nu]dp.
\end{aligned} \tag{6}$$

According to evolutionary game theory, the access strategy θ is an evolutionary stable strategy (ESS) if for any $\hat{\theta} \neq \theta$, there exists some $\tau' \in (0, 1)$ such that $\bar{U}_m < \bar{U}_n$ for all $\tau \in (0, \tau')$. In other words, mutants adopting distinct access probability will eventually become extinct if θ is ESS, which is an important factor for the resilience of CRN.

With (3), the ESS of the evolutionary access game exists if

$$\int_0^1 [\theta(p) - \hat{\theta}(p)]p[(\delta + \nu)A \exp(-Bf(\tau\hat{\theta} + (1 - \tau)\theta)) - \nu]dp > 0. \tag{7}$$

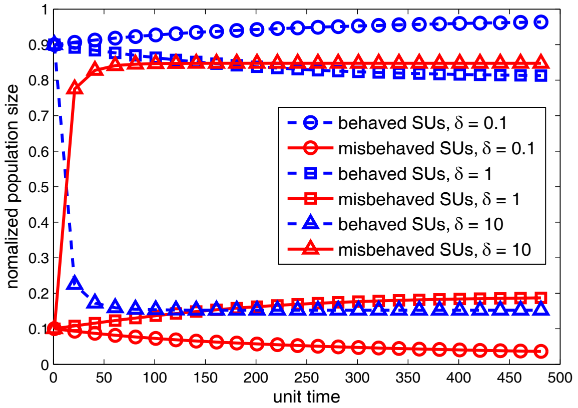

Fig. 2. Evolutionary access dynamics in underlay sharing paradigm. Non-mutants represent the SUs adopting initial access probability $\widetilde{p}$, and mutants represent the SUs transmitting for sure. The system parameters are set to be $\tau = 0.1$, $\alpha = 4$, $\lambda_{PT} = 10^{-5}$, $\eta_{PR} = 3$, $\epsilon_{PR} = 0.05$, $r_{PT} = 15$, $P_{PT} = 0.3$, $\lambda_{SU} = 10^{-3}$, $\eta_{SU} = 3$, $\epsilon_{SU} = 0.1$, $r_{SU} = 10$, $P_{SU} = 0.1$, $N = 10^{-9}$, $\mu = 1$ and $\nu = 5$.

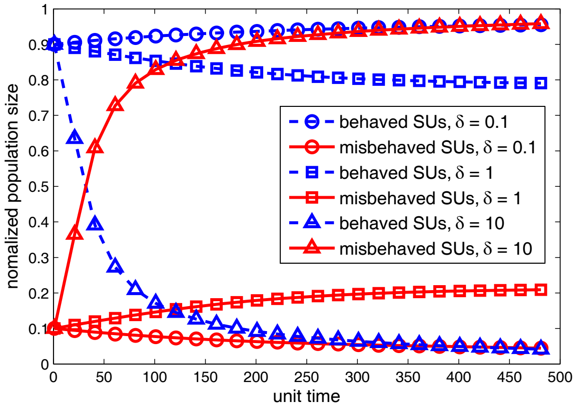

Fig. 3. Evolutionary access dynamics in interweave sharing paradigm, where $\widetilde{p} = 0$ and the system parameters are the same as Fig. 2.

The ESS therefore plays an essential role in understanding robustness of CRN protocol design since SUs are able to regulate the abnormal behaviors via self-enforcement if there are less than τ portion of misbehaved SUs.

IV. PERFORMANCE EVALUATION

To investigate the evolution of access probability, replicator dynamics are leveraged to update the access probability based on the experienced payoff. The subpopulation of a certain access probability (e.g., p_j) changes with a rate proportional to the difference between the payoff of that access probability and the average payoff of the total population. The replicator dynamic equation is expressed as

$$\frac{\partial \theta_t(p_j)}{\partial t} = \mu \theta_t(p_j) \left[\bar{U}_j(p_j|\theta_t) - \bar{U}(\theta_t, \theta_t) \right], \tag{8}$$

where θ_t is the normalized population distribution of SUs at time t and μ is some positive constant which is associated with the rate of convergence. In evolutionary access game, μ can be interpreted as the sensitivity to spectrum status.

Due to misbehaved access, initially we assume that each SU may transmit with probability τ regardless of the interference confinement from PUs. As shown in Fig. 2, when the incentive is small ($\delta = 0.1$), the misbehaved SUs are able to return to the initial access strategy via self-enforcement since the mutants do not benefit from misbehaved access. However, when the incentive grows, the self-enforcement fails to regulate the misbehaved SUs. When $\delta = 1$, at the steady state approximately 20 percent of SUs behave abnormally with different access strategy, and when $\delta = 10$, the mutants become the dominating species since the original access strategy is not attractive for survivability. Note that although population equilibrium occurs at steady state, the resulting aggregated interference from SUs may cause severe impacts on PRs and further deteriorates the network, as ecology and evolution suggests [10].

Similar phenomenon can be found in interweave paradigm as shown in Fig. 3 because high incentive nurtures the growth of misbehaved SUs to transmit concurrently with PUs.

V. CONCLUSION

Given that the existence of misbehaved SUs is inevitable in CRN, this letter studies the resilience of CRN against misbehaved SUs. By modeling misbehaved SUs as mutants with distinct access strategy, we could analyze the dynamics of access strategy in CRN via evolutionary access game. When the incentive of successful secondary transmission is small, the misbehaviors can be regulated via self-enforcement; however, when the incentive is high, SUs misbehave despite the sharing rules and behaved SUs will be extinct, which deteriorates the survivability of both underlay and interweave CRN. The operator shall carefully design the incentive or utility to ensure SUs to comply the sharing rules, otherwise, adversary may exploit equivalent denial-of-service attacks to collapse the CRN.

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